Gas thickness and the radial spin current

What the observations resolve in NGC 2403

Between **10.93 and 16.89 kpc**, thirteen rotation measurements overlap both the H I column map and a published radial thickness profile. If neutral hydrogen supplies the entropy density in the original ESFD galaxy equation, the weighted radial spin current must increase **3.96-fold** between those endpoints. Thickness gives the reconstruction a physical depth. Polarimetry leaves the microscopic direction undetermined.

Measurement	Result	What enters the calculation
Disk orientation	Inclination 63°, receding axis at position angle 124°	Constrains the orientation of bulk rotation. A ring's angular-momentum normal has zero radial projection within its local plane.
Radio polarization	28 ± 5 mJy integrated, 30–50 μJy per beam typical after smoothing	Constrains the sky projection of the ordered magnetic field. The disk signal is faint.
Neutral disk thickness	Published Gaussian scale height 0.08 to 0.68 kpc at 0.64 to 17.61 kpc	The authors infer the heights from H I kinematics and vertical hydrostatic balance in a fitted potential.
Extraplanar hydrogen	Flux-weighted exponential scale height 0.93 kpc	Li et al. model a distinct fountain component out to 12.5 kpc. The main-disk calculation uses the disk heights above.

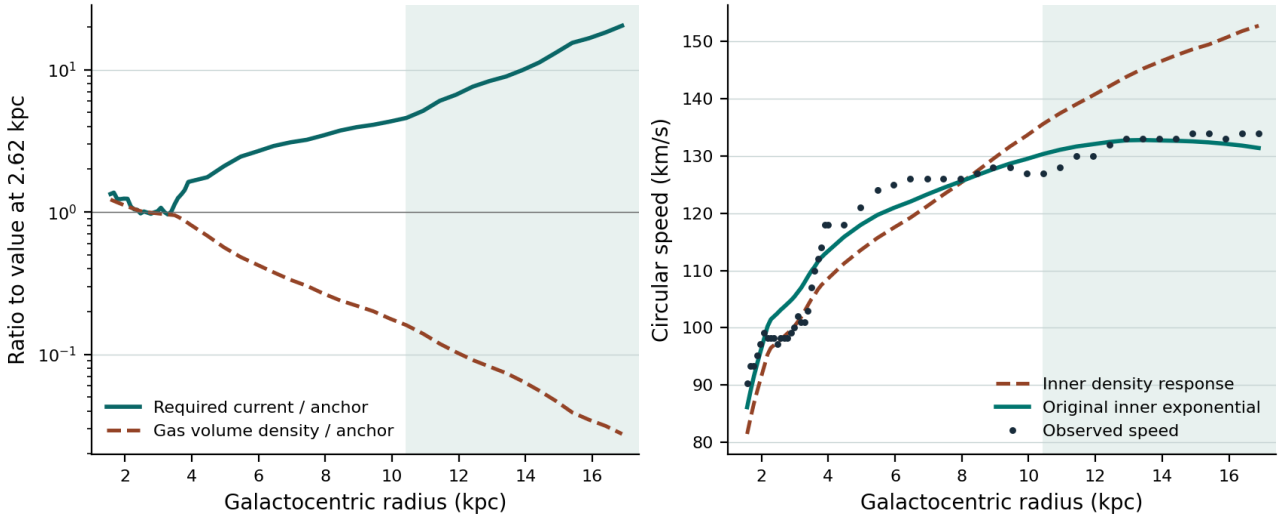
Heald et al. report **21 ± 4°** for the optical spiral pitch. Their polarization data leave the radial sign of the microscopic spin expectation open. Linear polarization has a 180° ambiguity. Faraday rotation measures a line-of-sight field weighted by free electrons. Neither supplies the directed spin density in $J = \varepsilon S u/2$ without an independently calibrated material response. A spin temperature constrains level populations, and opposite magnetic-sublevel imbalances can have the same temperature and entropy.

Heald et al., Table 1 and NGC 2403 notes · Mancera Piña et al., disk dynamics · Authors' radial tables · Li et al., extraplanar H I.

Converting the measured column into volume entropy

In the original stationary spherical weak-gravity reduction, $Q = \beta_s s J_r/c^2 = (v^2 - v_b^2)/(4\pi G r)$. The galaxy residual determines Q. To obtain s, identify it with the midplane neutral-hydrogen hyperfine entropy. Write $L = \sqrt{(2\pi)}h$ for a Gaussian of standard deviation h, $N = \Sigma_{HI}/m_H$ and $s = N k_B H/L$, where H is dimensionless entropy per atom. This uses a disk midplane tracer for the radial field in a spherical effective calculation.

Braun's 53 ± 15 K sightline at 2.620 kpc fixes the entropy scale under equal occupation of the triplet magnetic sublevels. Here h = 0.1424 kpc, L = 0.3570 kpc, and the inferred midplane entropy is $\mathbf{1.486 \times 10^{-17}\ J\ K^{-1}\ m^{-3}}$. The rotation inversion then gives $\mathbf{\beta_s J_r/c^2 = 4.044 \times 10^{15}\ kg\ K\ m/J}$. An independent oriented spin density would separate β_s from J_r. The anchor uses a pencil-beam temperature, an annular H I column and an interpolated rotation residual at the same radius.



The shaded region starts beyond the last training radius. The left panel reconstructs the source from the measured speed. On the right, both model curves use the inner data alone. All calculations retain the archived baryonic mass contributions. The thickness tables use a distance of 3.2 Mpc for NGC 2403. Their radii and heights have been scaled together to the project distance of 3.16 Mpc.

Braun, 21-cm absorption, Table 3 · HALOGAS map release · SPARC rotation curves · Thickness supplement.

What drives the rise inside the SFD equation?

Across the same thirteen outer rings the fitted log slopes are −0.599 for Q, −2.941 for the H I column, and +0.798 for the scale height. At constant per-atom entropy this gives a volume-density slope of −3.739 and a required current slope of **+3.140**. The alternative halo-concentration prior gives +3.121. Drawing the reported speed errors independently yields +2.97 to +3.30 for the primary fit, conditional on the baryonic model, geometry and H I profile. The former +3.470 result applied to the current per depth over a longer interval reaching 20.87 kpc.

> $J_r/J_{r,0} = (Q/Q_0)\,(\Sigma_0/\Sigma)\,(h/h_0)\,(H_0/H)$.

Keeping the current constant would require h to fall with slope −2.34, whereas the measured gas model flares. If the same atoms also supply the spin tensor, J_r scales with number density times signed radial spin fraction P. The corresponding P rises with slope +6.88. The largest $|P/P_0|$ is 738 in the measured donor profile. The bound $|P| \le 1$ implies $|P_0| \le 0.00136$ under this common-carrier and near-maximal-entropy approximation. Significant sublevel polarization would require recomputing H.

A source constraint in the original theory

The stationary SFD equation reads $\beta_s \operatorname{div} J = 2\gamma_s \Delta s - U'(s)$. The measured endpoint ratio of $r^2\beta_s J_r$ is **9.47**. Its nominal value increases across all twelve adjacent shells. The shell mean of $(2\gamma_s \Delta s - U')/c^2$ ranges from 3.11×10^{-4} to 8.61×10^{-4} kg K/J under the adopted entropy normalization. These are finite differences of the flux, with shared rings and correlated systematics. They supply a target for the material dynamics.

With a regular center, the enclosed mean is $\langle U' \rangle = 6\gamma_s s'/r - 3c^2 Q/(rs)$. The fitted entropy decreases across every outer interval while Q is positive. Positive stiffness therefore requires a negative enclosed mean potential derivative wherever s' is negative. Many potentials and material spin states satisfy this condition. The sign and shell strengths narrow the original equations without identifying a unique microscopic source.

Original ESFD direct action and SFD entropy variation, as reproduced in the preceding studies · Disk thickness model and its halo priors. Numerical checks and shell-by-shell results accompany the paper.

Transferring the same weights to other galaxies

Every target uses the NGC 2403 normalization at 2.62 kpc. Let $x = (\Sigma/h)/(\Sigma_0/h_0)$. Constant current predicts $Q/Q_0 = x$, and constant spin fraction in the same gas predicts x^2. A third test allows $J/J_0 = x^p$, with $p = \mathbf{-0.9258}$ fitted solely to 38 NGC 2403 measurements at 1.57–10.43 kpc. It predicts $Q/Q_0 = x^{0.0742}$. This exponent summarizes the donor data. The original equations supply the force and circular speed. The comparison imposes no outer speed or asymptotic slope.

Prescription	NGC 2403 outer, 13 rings	Two other spirals, 39 rings	Ten dwarfs, 307 rings
Baryons alone	71.27	40.29	20.34
Original inner exponential	1.62	10.89	28.81
Fixed H I column weight	38.36	18.52	10.19
Fixed volume current	61.13	33.20	13.28
Fixed spin fraction	70.40	38.91	18.58
Inner density response	13.80	12.36	18.88

Entries give mean per-galaxy speed RMSE in km/s. The density response improves on baryons in both comparison spirals and six dwarfs. The simpler column weight improves nine dwarfs. On the 137 outer dwarf rings the density response gives 21.34 km/s, the column weight 7.86, and baryons 24.24. Only one outer comparison-spiral ring has all three inputs, so their column reports the full joint radial coverage.

NGC 2403's outer speed slope is +0.090. The fitted density response predicts +0.242 and the original exponential +0.004 on the held rings. The exponential reproduces this finite near-plateau while ultimately decaying, and transfers poorly to these dwarfs. The density response supplies a partial explanation for the inferred rise in the donor, but does not establish a common spin law or flat speeds at arbitrarily large radius.

SPARC · Oh et al., resolved mass models · Hunter et al., dwarf H I distributions · Mancera Piña et al..