ESFD and SEFD from first principles

The spin–field action, its entropy extension, and the equations that predict a galaxy's rotation curve.

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1 Where the equations begin

Christopher Grenke's starting question was prompted by a spinning top. Its rotation changes as it wobbles and slows. Could the local electromagnetic environment and the spins of the particles making up an object also contribute to how it moves? In the June manuscript the first mathematical step is to describe matter and electromagnetism at the *same place and time*. Only then can their interaction be written down.

Three meanings of spin. A top turning about its axis has bulk angular momentum. An electron has intrinsic angular momentum, a quantum property present even without a visible orbit or rotation. A material also has a *net polarization* when the spins of many constituents preferentially align. Imagine two ensembles with the same number of electrons. One has equal populations pointing in opposite directions. The other has a small surplus in one direction. The second can have nonzero average spin polarization even if the object is sitting still. ESFD needs a material model to pass between these meanings.

Which names belong to which equations? The umbrella is **Electromagnetic Spin–Field Dynamics (ESFD)**. A June 3, 2025 manuscript also uses *Spin–Field Saturation Dynamics* in its title and keeps the acronym ESFD. The June 3 consolidation calls the galaxy companion **Spin–Entropy Field Dynamics (SEFD)**. The association with baryonic galaxy curves is the basis for interpreting "SFFD" as SEFD here. The archive preserves more than one proposed field action. Each one below is identified at its source.

How to read the notation. A field assigns something to every point. Temperature assigns a number. A wind field assigns a speed and direction. A tensor assigns components that transform together when you change coordinates. Write a point as x^μ. Index 0 labels time and indices $1, 2, 3$ label space. Greek indices run over all four. A repeated upper and lower index means sum over its four possible values. An index left over tells you which component of the answer you are calculating.

What an action does. A Lagrangian density $\mathcal{L}$ gives the local rule for the fields. Integrate it over space and time to get the action $\mathcal{A}$. If you slightly change a candidate field history while fixing its boundary data, the physical field equations require the first-order change in the action to vanish. This is written $\delta\mathcal{A} = 0$. It gives a set of equations for the fields. Initial conditions, source terms and measured constants then select a particular physical solution.

Units used here. For the field actions, $c = \hbar = 1$ and the spacetime signature is $(+,-,-,-)$. This convention measures time and distance in compatible units. In the star and galaxy applications, units are stated explicitly. Subscripts on reused letters identify the source and meaning of each constant.

2 Maxwell's field and the spin of matter

From a potential to a field. The electromagnetic potential A_μ contains an electric potential and a magnetic vector potential. The change of one component of the potential along another coordinate gives the electromagnetic field tensor,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \qquad \partial_\mu = \frac{\partial}{\partial x^\mu}.$$

The minus sign makes $F_{\nu\mu} = -F_{\mu\nu}$, so the diagonal entries are zero. A four-by-four array now has six independent entries. Three describe the electric field **E**, and three describe the magnetic field **B**. For example, $F_{12} = \partial_1 A_2 - \partial_2 A_1$ measures a local circulation of the vector potential in the 1–2 plane and encodes a magnetic component. The exact signs assigned to the electric and magnetic components depend on the coordinate convention. Changing A_μ by the gradient of a scalar leaves $F_{\mu\nu}$ unchanged.

In SI units, the ordinary field with charge density ρ_q and spatial current **j** satisfies

$$\begin{aligned} \nabla \cdot \mathbf{E} &= \rho_q/\epsilon_0, \quad & \nabla \cdot \mathbf{B} &= 0, \\ \nabla \times \mathbf{E} &= -\partial_t \mathbf{B}, \quad & \nabla \times \mathbf{B} &= \mu_0 \mathbf{j} + \mu_0 \epsilon_0 \, \partial_t \mathbf{E}. \end{aligned}$$

Here ϵ_0, μ_0 are the vacuum permittivity and permeability. The first row says charges source electric fields, and net magnetic flux through a closed surface is zero. The second says that a changing magnetic field induces a circulating electric field, and both current and a changing electric field produce magnetic circulation. A dot with ∇ denotes divergence. A cross with it denotes curl.

From a matter field to a spin tensor. The manuscript represents a spin-½ matter field such as an electron by a Dirac spinor ψ. It has four complex components. The following definitions convert it into the local antisymmetric tensor that appears in the interaction,

$$\bar{\psi} = \psi^\dagger \gamma^0, \qquad \sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu], \qquad S_{\mu\nu} = \frac{1}{2}\bar{\psi}\sigma_{\mu\nu}\psi.$$

Here $i^2 = -1$, the dagger means complex conjugate and transpose, and $[P, Q] = PQ - QP$. The γ^μ are matrices that make the spinor equation consistent with spacetime geometry. They obey $\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}\mathbf{1}$, where **1** is the identity matrix. The metric $g_{\mu\nu}$ measures intervals and raises or lowers indices. The bar on ψ is the adjoint used to build quantities with a defined transformation under changes of frame.

What the tensor pairing measures. Since both tensors are antisymmetric, their contraction reads

$$\begin{aligned} S_{\mu\nu}F^{\mu\nu} = 2(&S_{01}F^{01} + S_{02}F^{02} + S_{03}F^{03} \\ &+ S_{12}F^{12} + S_{13}F^{13} + S_{23}F^{23}). \end{aligned}$$

This gives one number at that point. The factor two counts the two index orders. In a magnetic rest-frame example it is sensitive to the projection of the magnetic polarization along the applied field. Turning the polarization around reverses the direct interaction for fixed field. Ordinary magnetic dipoles already display orientation-dependent energies. A new ESFD contribution must therefore be specified relative to the known electromagnetic response.

3 ESFD in the direct spin–field manuscript

Section 3 of the June 3 manuscript supplies all five terms explicitly. Here α_J is the manuscript's α, and λ_S is its λ. The subscript J keeps this convention identifiable later.

$$\mathcal{A}_J = \int d^4x\,\sqrt{-g}\left[\frac{R-2\Lambda_{\text{cc}}}{16\pi G} + \bar{\psi}(i\gamma^\mu D_\mu - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \alpha_J S_{\mu\nu}F^{\mu\nu} - \frac{\lambda_S}{4}(S_{\rho\sigma}S^{\rho\sigma} - \Phi_0^2)^2\right].$$

1. Spacetime geometry. R is the Ricci scalar, a contraction of spacetime curvature. G is Newton's gravitational constant, and Λ_{cc} is a cosmological constant. The volume element $d^4x\,\sqrt{-g}$ counts physical spacetime volume, with $g = \det(g_{\mu\nu})$. This version includes Einstein gravity from the start. Deriving gravity from spin alone is a separate claim made in the proof draft.

2. Matter and electric charge. The Dirac term tells the spin-½ matter field how to evolve. m is its mass, and the June manuscript uses $D_\mu = \nabla_\mu - iqA_\mu$. Here q is its signed electric charge and ∇_μ is the derivative that accounts for the changing spacetime frame. This is the usual minimal electromagnetic coupling of charged matter.

3. Electromagnetism. $-F_{\mu\nu}F^{\mu\nu}/4$ supplies the vacuum field dynamics. In local natural units it equals $(|\mathbf{E}|^2 - |\mathbf{B}|^2)/2$. The electromagnetic **energy density** is $(|\mathbf{E}|^2 + |\mathbf{B}|^2)/2$. A minus sign in the Lagrangian does not imply negative magnetic energy.

4. Direct spin response. $\alpha_J S_{\mu\nu}F^{\mu\nu}$ couples the matter polarization to the field beyond minimal coupling. The sign sets the favored relative orientation in a given energy convention, and its magnitude sets the response strength. A numerical prediction requires a specified matter species or medium and a calibrated coupling.

5. Saturation. Define $X = S_{\rho\sigma}S^{\rho\sigma}$. The potential $V_{\text{sat}} = \lambda_S(X - \Phi_0^2)^2/4$ assigns its lowest value at $X = \Phi_0^2$ for $\lambda_S > 0$. The reference scale Φ_0 sets that preferred value, and λ_S sets its stiffness. Departures of finite size cost finite potential. An absolute upper bound or collapse threshold requires a further dynamical argument.

4 Opening the brackets and obtaining the equations of motion

The matter and saturation terms can be expanded one step further,

$$\begin{aligned}\bar{\psi}(i\gamma^{\mu}D_{\mu}-m)\psi &= i\bar{\psi}\gamma^{\mu}\nabla_{\mu}\psi + qA_{\mu}\bar{\psi}\gamma^{\mu}\psi - m\bar{\psi}\psi,\\ X &= \tfrac{1}{4}(\bar{\psi}\sigma_{\rho\sigma}\psi)(\bar{\psi}\sigma^{\rho\sigma}\psi),\\ -V_{\text{sat}} &= -\tfrac{\lambda_S}{4}X^2 + \tfrac{\lambda_S\Phi_0^2}{2}X - \tfrac{\lambda_S\Phi_0^4}{4}.\end{aligned}$$

These are, respectively, propagation, coupling to electric charge, and rest mass in the first line. The final line displays the spin self-interactions and a constant. A constant in the action can contribute to the vacuum stress when gravity is varied.

Vary the potential A_{μ}. With the charge sign convention of the June manuscript and a torsion-free covariant derivative, the result is

$$\begin{aligned}H^{\mu\nu} &= F^{\mu\nu} - 2\alpha_J S^{\mu\nu},\\ \nabla_{\nu}H^{\mu\nu} &= J^{\mu}, \qquad J^{\mu} = q\bar{\psi}\gamma^{\mu}\psi,\\ \partial_{\lambda}F_{\mu\nu} + \partial_{\mu}F_{\nu\lambda} + \partial_{\nu}F_{\lambda\mu} &= 0.\end{aligned}$$

J^{μ} contains electric charge density and current. $H^{\mu\nu}$ collects the ordinary field and the polarization response. A varying polarization contributes an effective source. The last identity follows from the definition of F and contains Faraday induction and the magnetic divergence law. The order of the indices matters because the tensors are antisymmetric.

Vary the adjoint matter field $\bar{\psi}$. Treating $S_{\mu\nu}$ as the stated bilinear gives

$$\left[i\gamma^{\mu}D_{\mu} - m + \frac{\alpha_J}{2}\sigma_{\mu\nu}F^{\mu\nu} - \frac{\lambda_S}{2}(X-\Phi_0^2)S^{\mu\nu}\sigma_{\mu\nu}\right]\psi = 0.$$

The applied field acts back on spin. The last term is the feedback of the spin distribution on itself. It follows by differentiating both $V_{\text{sat}}(X)$ and $X(\bar{\psi},\psi)$. The short Dirac equation printed in the draft omits this derivative. This displayed equation is the variation of the specified action.

Vary the metric. In the metric, torsion-free version, $G_{\mu\nu} + \Lambda_{\text{cc}}g_{\mu\nu} = 8\pi G\,T_{\mu\nu}^{\text{total}}$. Here $G_{\mu\nu}$ is the Einstein curvature tensor and $T_{\mu\nu}^{\text{total}}$ collects energy, momentum and stress from the other terms. Einstein–Cartan variants additionally vary an independent connection Γ.

5 The proof draft and the exponential ESFD branch

The shorter proof-draft action. The opening of the project PDF *Proof of the Electromagnetic Spin–Field Dynamics Theory* gives a vacuum Lagrangian using the vacuum permeability μ_0 and an interaction coefficient κ_P,

$$\mathcal{L}_P = -\frac{1}{4\mu_0} F_{\mu\nu}F^{\mu\nu} + \frac{\kappa_P}{2} S^{\mu\nu}F_{\mu\nu} - V_P(S).$$

For an independently varied spin tensor and a potential with no spin derivatives, its flat-space equations are $\partial_\mu(F^{\mu\nu}/\mu_0 - \kappa_P S^{\mu\nu}) = 0$ and $\partial V_P/\partial S^{\mu\nu} = \kappa_P F_{\mu\nu}/2$, with contraction over antisymmetric index pairs understood. Here the spin response is algebraic. The draft sketches a long-range potential proportional to $-\kappa_P^2\langle S_A S_B\rangle/r$ between bodies A, B at separation r. To obtain Newton's law one must derive that range, an attractive coupling proportional to both masses, and the observed value of G.

The exponential branch. The merged June formulation puts spin into the coefficient of the electromagnetic term. Its kinetic normalization was written as one. The explicit factor f_Φ^2 below restores units when Φ is dimensionless,

$$\mathcal{A}_{\text{exp}} = \int d^4x\sqrt{-g}\Big[\frac{R(\Gamma) - 2\Lambda_g}{16\pi G} - \tfrac{1}{4}e^{-\xi\Phi}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}f_\Phi^2 g^{\mu\nu}\partial_\mu\Phi\,\partial_\nu\Phi - V_\Phi(\Phi) + \mathcal{L}_{\text{matter}}\Big].$$

Γ determines parallel transport, Λ_g is the cosmological term of this version, $\Phi = \Phi_{\text{EM}}$ is its scalar, and ξ sets how strongly it changes the field coefficient. The kinetic term permits spatial and temporal variation. V_Φ specifies which values are favored. The recovered composite prescription gives $\Phi = \kappa_{\text{EM}}^2 X$. The June direct manuscript elsewhere used $\Phi_{\text{EM}} = \sqrt{X}$ on the real branch $X \geq 0$. The two definitions have different units and responses.

Write $F^2 = F_{\mu\nu}F^{\mu\nu}$. For small $|\xi\Phi|$, the electromagnetic term becomes $-F^2/4 + (\xi\kappa_{\text{EM}}^2/4)XF^2 + \cdots$ with the composite prescription. Reversing all spins changes the sign of the direct $S : F$ term and leaves XF^2 unchanged. That gives a physical way to tell the two proposed interactions apart.

If Φ is **independent**, its equation is $f_\Phi^2\,\Box\,\Phi + V_\Phi' = (\xi/4)e^{-\xi\Phi}F^2$ when matter has no other explicit dependence on Φ. $\Box = \nabla_\mu\nabla^\mu$ is the wave operator. If Φ is **defined by** X, variations must pass through the matter field or use an explicit constraint. Treating a composite scalar as a second free field changes the theory.

The schematic in the re-evaluation. Section 1.1 of the *Re-Evaluation* PDF records $\mathcal{A} = \int d^4x\,[\mathcal{L}_{\text{EH}} + \mathcal{L}_{\text{EM}} + \mathcal{L}_{\text{spin}} + \chi\mathcal{L}_{\text{spin-EM}}]$. Its four blocks name curvature, electromagnetism, intrinsic spin dynamics and their coupling. The coefficient χ here multiplies an interaction. The expression leaves the spin functions unevaluated. The June direct action supplies one explicit choice for those blocks.

6 SEFD in the original spin–entropy consolidation

The June consolidation starts with a compact spin–electromagnetic Lagrangian, then adds three entropy terms. Let $s = \mathcal{S}(x)$, the proposed local entropy density, and write $U(s) = V(\mathcal{S})$ for its potential. Entropy density describes how much microscopic information is left unspecified by a macroscopic description per unit volume. To use it in a material or a galaxy one must choose what microscopic states are being counted and how to measure or compute s.

$$\begin{aligned}\mathcal{L}_{\text{SEFD}} = & -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} - \tfrac{\alpha_C}{2}S^{\mu\nu}F_{\mu\nu} + \mathcal{L}_{\text{matter}}(S^{\mu\nu},\psi) \\ & - \beta_s s\,\nabla_\mu J_s^\mu + \gamma_s g^{\mu\nu}(\partial_\mu s)(\partial_\nu s) - U(s), \\ J_s^\mu = & \tfrac{1}{2}\varepsilon^{\mu\nu\rho\sigma}S_{\nu\rho}u_\sigma.\end{aligned}$$

The first line is the consolidation's ESFD base. It leaves the matter term as a named function. It does not print an Einstein term or the June quartic separately on that line. Its direct interaction uses $-\alpha_C/2$ where the full June action uses $+\alpha_J$. With identical field units the coefficients of the tensor product match if $\alpha_J = -\alpha_C/2$. A numerical value of one should not be transferred unconverted to the other.

The spin current J_s^μ. u_σ is the local four-velocity of the material or chosen rest frame. The four-index ε is the oriented, antisymmetric spacetime volume tensor. Swapping two indices reverses its sign. Combining it with S and u gives a spin-direction quantity whose time component vanishes in that rest frame, since $u_\mu J_s^\mu = 0$. This is the archive's definition of the candidate current. Its conversion into a physical transport rate or thermodynamic entropy flux still needs material physics.

The coupling $-\beta_s s\,\nabla_\mu J_s^\mu$. A divergence measures the local balance of outward flow and accumulation. β_s weights the proposed coupling of entropy density to this balance. A uniform stationary current in flat space has zero divergence. Its direct local source for s then vanishes. The parameter β_s can carry units.

The gradient and potential. In a local inertial frame the gradient term is $\gamma_s[(\partial_t s)^2 - |\nabla s|^2]$. It assigns dynamics to changes in time and space. Its coefficient γ_s is unrelated to the Dirac matrices γ^μ. $U(s)$ determines preferred values and how strongly s returns after a disturbance. The consolidation does not specify a unique function U or calibrated values for the three couplings $\alpha_C, \beta_s, \gamma_s$.

7 Writing the direct ESFD and entropy terms together

One can put the five terms of the June ESFD action and the three entropy terms on a single page. Doing so requires a choice the consolidation leaves open. Here the compact matter sector is completed with the June Dirac field and saturation potential, the June Einstein term is retained, and the differing dipole coefficients are matched by $\alpha_J = -\alpha_C/2$. These choices specify the following **combined direct-action example**. The separate historical actions remain those printed earlier.

Abbreviate the spinor product and the curl of the potential by

$$Q_{\mu\nu} = \bar{\psi}\sigma_{\mu\nu}\psi, \qquad K_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu.$$

Thus $S = Q/2$, $F = K$, $X = Q_{\mu\nu}Q^{\mu\nu}/4$, and $J_s^\mu = \varepsilon^{\mu\nu\rho\sigma}Q_{\nu\rho}u_\sigma/4$. With these substitutions the action is

$$\begin{aligned}\mathcal{A}_{\text{joined}} = \int d^4x\sqrt{-g}\,\Big[&\frac{R - 2\Lambda_{\text{cc}}}{16\pi G} + i\bar{\psi}\gamma^\mu\nabla_\mu\psi + qA_\mu\bar{\psi}\gamma^\mu\psi - m\bar{\psi}\psi \\ &- \tfrac{1}{4}K_{\mu\nu}K^{\mu\nu} + \tfrac{\alpha_J}{2}Q_{\mu\nu}K^{\mu\nu} \\ &- \tfrac{\lambda_S}{4}\big(\tfrac{1}{4}Q_{\mu\nu}Q^{\mu\nu} - \Phi_0^2\big)^2 \\ &- \tfrac{\beta_s}{4}s\,\nabla_\mu(\varepsilon^{\mu\nu\rho\sigma}Q_{\nu\rho}u_\sigma) \\ &+ \gamma_s g^{\mu\nu}(\partial_\mu s)(\partial_\nu s) - U(s)\Big].\end{aligned}$$

Read from the top down. Geometry sets the local spacetime frame. The matter field moves through it, carries charge and has rest mass. The electromagnetic field propagates and acts on the spin carried by that matter. The spin self-interaction assigns a cost to deviations from its preferred invariant. The entropy field couples to the divergence of the spin current built from the same matter, changes through space and time, and responds to its own potential.

What must be prescribed to solve it. Specify $U(s)$, the matter and charge content, how u^μ is obtained, all couplings, the external fields, and the initial and boundary conditions. Prescribe u^μ as a material background or give the material its own dynamics. Use one choice consistently when varying the action. Varying this joined model with respect to the matter field gives terms from the entropy interaction in addition to the direct ESFD equation. The separate galaxy-density prescription must still be justified. Appending a fitted rotation curve or an empirical stellar cutoff as another local term would not supply that justification.

8 What the entropy equation actually drives

Hold the current fixed while varying s, take the couplings to be constants, and fix the boundary variation. The Euler–Lagrange equation is

$$2\gamma_s \,\Box\, s + U'(s) = -\beta_s \nabla_\mu J_s^\mu .$$

The prime on U means its derivative with respect to s. In locally flat spacetime $\Box\, s = \partial_t^2 s - \nabla^2 s$ in natural units. The first term lets a disturbance travel. The second term is the restoring response set by the potential. The right side specifies where the spin current drives the entropy field. If the current and material velocity also vary, equations for the matter and velocity must be solved with this one.

For example, near a stable value s_0, write $U(s) = U(s_0) + K(s - s_0)^2/2$ with $K > 0$. Then $U' = K(s - s_0)$. A homogeneous disturbance in a region with zero current divergence obeys $2\gamma_s\, d^2(s - s_0)/dt^2 + K(s - s_0) = 0$. For $\gamma_s > 0$ it oscillates with $\omega^2 = K/(2\gamma_s)$. This example displays how a characteristic frequency can arise once the response coefficients are supplied. It gives no numerical resonance frequency without those inputs.

From four-dimensional field dynamics to a galaxy source. In a slow, stationary galaxy model the consolidation *also proposes*

$$\rho_{\text{eff}} \simeq \frac{\beta_s}{c^2} \nabla \cdot (s \mathbf{J}_s).$$

Here ρ_{eff} is the mass density that would produce an additional gravitational acceleration in the Newtonian model. $\mathbf{J}_s$ is the spatial part of the current and c is the speed of light restored for the archived mass conversion. β_s denotes the coefficient as used in the archived prescription. A physical SI prediction needs an explicit unit and normalization map from the action to this density.

The distinction is visible by multiplying out the derivative,

$$\nabla \cdot (s \mathbf{J}_s) = s\, \nabla \cdot \mathbf{J}_s + \mathbf{J}_s \cdot \nabla s.$$

The source in the entropy *field equation* contains the four-divergence of J_s^μ. The galaxy *density prescription* contains the spatial divergence of the product $s\mathbf{J}_s$. They are different operations. The density law is an extra assumption of the galaxy branch. A derivation would have to produce the same gravity source from the full fields and stress tensor with physically specified boundary conditions.

9 From spherical flux to the original galaxy curve

Imagine a sphere of radius r centered on a galaxy. Define $A_r(r) = (\beta_s/c^2)s(r)J_s^r(r)$ as the outward weighted flux per unit area in this model. This A_r is unrelated to the electromagnetic potential A_μ. A sphere has area $4\pi r^2$. Neighboring shells enclose different areas even if the outward flux per unit area stays the same.

$$\begin{aligned}\rho_{\text{eff}}(r) &= \frac{1}{r^2}\frac{d}{dr}[r^2 A_r(r)] = A_r'(r) + \frac{2A_r(r)}{r} \\ &= \frac{\beta_s}{c^2}\left(s' J_s^r + s(J_s^r)' + \frac{2sJ_s^r}{r}\right).\end{aligned}$$

The June chat's expanded radial line includes the two product derivatives and omits the final area term. The correction matters physically. An outward radial flux constant in magnitude over a shell has a nonzero divergence $2A_r/r$ there. Primes on radial quantities here mean differentiation with respect to distance.

Choose the archive's exponential **flux** $A_r = Ce^{-r/r_s}$. C is the central amplitude of this profile and $r_s > 0$ is the distance over which the flux falls by a factor e. For this choice C has units of mass per area. For $r > 0$ and no point mass from this source at the origin, spherical integration gives

$$\begin{aligned}\rho_{\text{eff}}(r) &= Ce^{-r/r_s}\left(\frac{2}{r} - \frac{1}{r_s}\right), \\ M_{\text{eff}}(< r) &= \int_0^r 4\pi r'^2 \rho_{\text{eff}}(r')\, dr' = 4\pi C r^2 e^{-r/r_s}, \\ v_{\text{pred}}(r) &= \sqrt{v_b^2(r) + 4\pi G C r e^{-r/r_s}}.\end{aligned}$$

$M_{\text{eff}}(< r)$ is the extra mass *inferred* inside radius r. For a circular orbit the inward acceleration is v^2/r. The added spherical acceleration is $GM_{\text{eff}}(< r)/r^2$. This explains why the **squared speeds** add to the measured-baryon contribution v_b^2. The disk used to compute v_b can itself be flattened.

For positive C the added squared speed peaks at r_s and declines thereafter. The effective density changes sign at $2r_s$, and the total enclosed effective mass tends to zero at infinite radius. The density also diverges as $1/r$ at the center, although its volume integral near the center is finite. These are properties of the literal trial profile.

A different release assumes an exponential **density** $\rho = \rho_0 e^{-r/r_s}$. Its enclosed mass is $4\pi\rho_0 r_s^3[2 - e^{-x}(x^2 + 2x + 2)]$ with $x = r/r_s$. This version stays positive and has finite positive asymptotic mass for $\rho_0 > 0$. Substituting it for the divergence would change the rotation curve.

10 The revised galaxy equation paired with baryons

The November master, equation 57, uses a revised radial law. It uses each galaxy's visible matter distribution to predict its inner contribution and assigns the additional contribution a core and a flat outer speed,

$$v_{\mathrm{pred}}(r) = \sqrt{v_b^2(r) + \sqrt{GM_b a_\star}\,\frac{r^2}{r^2+\eta^2 R_d^2}},$$
$$v_\infty^4 = GM_b a_\star, \qquad r_c = \eta R_d.$$

The baryons. Baryons here mean the ordinary matter counted in the stellar and gas mass model. M_b is its total mass. $v_b(r)$ records how that matter is *distributed*, which also affects the local acceleration. In the SPARC tables the reference gas, stellar disk and bulge contributions can use signed velocities. With that convention,

$$v_b^2 = v_g|v_g| + \Upsilon_d v_d|v_d| + \Upsilon_{\mathrm{bul}} v_{\mathrm{bul}}|v_{\mathrm{bul}}|.$$

$v_g, v_d, v_{\mathrm{bul}}$ are the tabulated gas, disk and bulge contributions. $\Upsilon_d, \Upsilon_{\mathrm{bul}}$ rescale the reference stellar predictions for the assumed mass per unit light. The project baseline uses 0.5 and 0.7 in the catalog's solar units. A galaxy without a bulge has zero bulge contribution. The sign records the direction of that component's gravitational acceleration. For positive entries $v|v| = v^2$.

The extra speed. r is orbital radius, R_d the measured stellar disk scale length, and η a fitted dimensionless multiplier giving the core radius r_c. The fitted acceleration scale $a_\star$ sets the speed normalization v_∞. At $r = r_c$ the radial fraction is $1/2$, so the extra *squared* speed is $v_\infty^2/2$. Far outside the core, the extra speed approaches v_∞. Inside the core it rises approximately linearly with r.

The corresponding spherical source. One potential, enclosed mass and density that produce the added speed are

$$\Phi_f(r) = \tfrac{1}{2} v_\infty^2 \ln[(r^2+r_c^2)/r_0^2],$$
$$M_f(<r) = \frac{v_\infty^2 r^3}{G(r^2+r_c^2)}, \qquad \rho_f(r) = \frac{v_\infty^2 (r^2+3r_c^2)}{4\pi G (r^2+r_c^2)^2}.$$

r_0 sets the arbitrary zero of potential. The radial weighted flux reproducing this density is $A_r = v_\infty^2 r/[4\pi G(r^2+r_c^2)]$. This construction gives a cored logarithmic potential and imposes the baryonic Tully–Fisher scaling $v_\infty^4 \propto M_b$. Taken to infinity its equivalent mass grows without bound. It needs an outer boundary or transition for a finite isolated mass. The project has not derived $a_\star$ or η from the microscopic couplings or measured sJ_s^r in these galaxies.

11 A measured galaxy, point by point

NGC 2403 shows what changes when the radial law changes. Each dot is the measured circular speed at one distance from the galaxy center. The curves use the same galaxy's gas and stellar inputs. The predictions below come from the stored whole-galaxy holdout. This galaxy's measured rotation speeds were withheld when estimating the shared coefficients used for its fold.

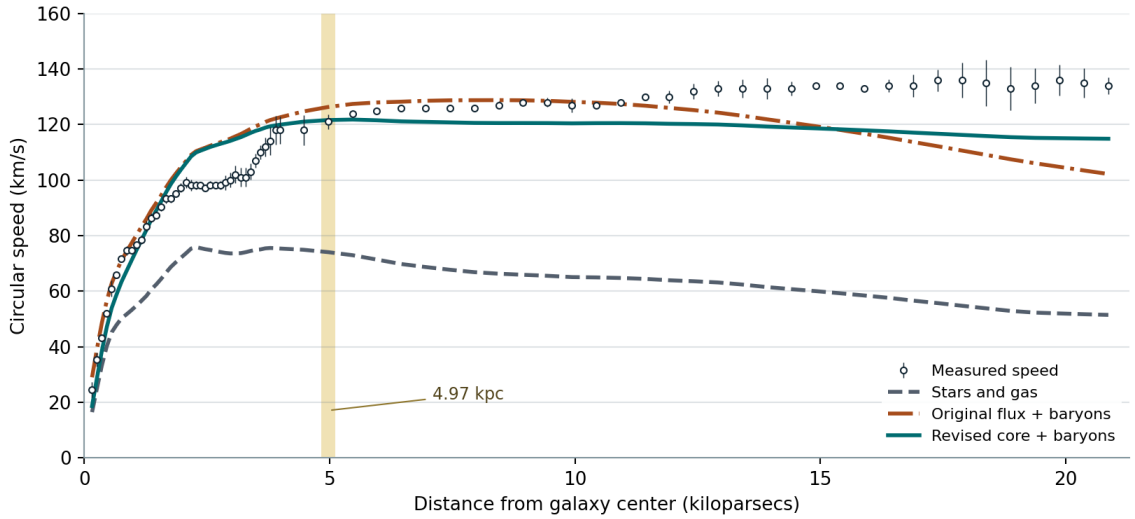

Figure 1. NGC 2403 rotation curve. The shaded vertical band marks the radius used in the worked example.

At **4.97 kiloparsecs** from the center, the measured speed is 121.00 ± 2.57 **km/s**. A kiloparsec is about 3,260 light-years. The baryons give **74.05 km/s**. The archival exponential spin–entropy flux plus baryons gives **126.34 km/s**. The revised cored law plus baryons gives **121.63 km/s**. The radial acceleration relation (RAR), a separately refitted empirical comparison, gives **113.30 km/s**.

The missing acceleration is easier to read as a speed squared. At this radius the observation requires an additional $121.00^2 - 74.05^2 \simeq 9{,}158\ (\mathrm{km/s})^2$, or about 95.7 km/s when expressed as the square root of that contribution. Adding 95.7 directly to 74.05 would give the wrong circular speed.

Across 131 galaxies, the median of each galaxy's root-mean-square speed error is 44.73 km/s for baryons alone, 19.82 for the original flux profile, 12.45 for the revised cored law and 13.11 for the refitted RAR. The revised law has smaller error in 67 of 131 paired comparisons with RAR. The median paired error difference is -0.003 km/s, with a galaxy-resampled 95% interval from -0.67 to $+0.68$ km/s. A difference of sample medians and a median of paired differences answer different questions. The data support a close empirical comparison. They do not identify a microscopic spin or entropy source.

12 Stellar rotation, strong fields and the several meanings of chi

The symbol χ labels different quantities in different project papers. Subscripts here identify what goes into each calculation.

1. Field times rotation. In the June direct manuscript,

$$\Omega = \frac{2\pi}{P}, \qquad \chi_{\text{rot}} = \frac{B\Omega}{\Lambda_{\text{tim}}}, \qquad \Lambda_{\text{tim}} = 6.28 \times 10^{14}\ \text{G rad s}^{-1}.$$

P is the time for one turn, Ω the angle turned per second, and B the adopted surface magnetic field in gauss. For $P = 1$ second and $B = 10^{14}$ gauss the index is about 1.00. That value is set by the chosen scale. In pulsar catalogs the common magnetic-dipole estimate is $B_{\text{tim}} \propto \sqrt{P\dot{P}}$, with $\dot{P}$ the change in period per unit time. Then $\chi_{\text{rot}} \propto \sqrt{\dot{P}/P} \propto 1/\sqrt{\tau_c}$ with characteristic age $\tau_c = P/(2\dot{P})$. Such a timing-only test reuses the same two measurements. Independent field or spin-polarization data would test additional physics.

2. The spin–entropy extension. The archived combined and entropy-only indices are

$$\chi_{\text{tot}} = \frac{\Omega(B + \kappa_s s)}{\Lambda_{\text{tim}}}, \qquad \chi_s = \frac{s\Omega}{\Lambda_s}.$$

κ_s converts the entropy variable to the units of the adopted magnetic field. Λ_s must have the units of $s\Omega$. A numerical threshold requires defining and measuring s in those units. The June proposal associates order-unity values with instability. The connection to an observed change of state requires a stability calculation and measurement.

3. The stellar energy budget. The project's *Testing* PDF uses an explicitly different estimate in SI,

$$\chi_E \simeq \frac{\frac{1}{2}I\Omega^2 + \frac{B_{\text{int}}^2}{2\mu_0}\frac{4\pi R_*^3}{3}}{0.6\,GM^2/R_*}, \qquad I \simeq 0.3MR_*^2.$$

M, R_*, I are stellar mass, radius and moment of inertia. The numerator adds rotation energy and estimated interior magnetic energy. The denominator estimates the magnitude of gravitational binding energy. B_{int} is a representative interior field in tesla, and μ_0 is the SI vacuum permeability. This Newtonian uniform-field estimate does not solve relativistic stellar stability.

4. The local strong-field ratio. The *Proof* PDF also uses $\chi_P = \mathcal{F}_{\text{rest}}/\mathcal{F}_{\text{crit}}$, an electric or magnetic field in a particle's rest frame divided by the appropriate critical field of the same kind. The *Re-Evaluation* uses χ as a coupling multiplying a term in an action. A measured value of any one of these quantities does not fix the others.

13 Counterpropagating waves and the superconducting extension

Two equal electromagnetic waves traveling in opposite directions provide an exact place to evaluate the Maxwell part. For equal, linearly polarized vacuum waves of amplitude E_0, angular frequency ω and wave number $k = \omega$ in natural units, set $C_\pm = \cos(kz \mp \omega t)$. The electric and magnetic amplitudes along perpendicular axes are $E_x = E_0(C_+ + C_-)$ and $B_y = E_0(C_+ - C_-)$. Consequently,

$$F_{\mu\nu}F^{\mu\nu} = -8E_0^2 C_+ C_- = -4E_0^2[\cos(2kz) + \cos(2\omega t)].$$

Each wave alone has zero invariant F^2. Their superposition has an interference term, including a modulation at twice the original frequency. The two ordinary waves account for it. A *third independently excitable mode* would need a dynamical variable, a coupling to this invariant and a measured response. In the project's illustrative collective-spin model $Z(\varphi) = e^{-g_w\varphi^2}$ multiplies the Maxwell term. Projecting onto a uniform spin mode in an externally maintained standing wave gives

$$\ddot{\varphi} + [m_0^2 + q_w \cos(2\omega t)]\varphi = 0, \qquad q_w = 2g_w E_0^2.$$

Dots mean time derivatives. m_0 sets the undriven angular frequency in these units, g_w the coupling strength and q_w the modulation. This equation periodically changes the restoring force. It has a parametric resonance near $\omega = m_0$ at weak drive. It is a specific model of an additional mode. Ordinary acoustic waves are pressure or elastic disturbances in matter, and a mechanical, electromagnetic or proposed spin resonance has to be identified by what oscillates.

For a superconducting material, the master also specifies a separate homogeneous free-energy model (equation 55). Use φ for the candidate neutral collective mode in this application and Ψ for the complex superconducting order parameter. Its magnitude squared $n = |\Psi|^2$ measures condensate density in the chosen normalization.

$$f = a(T)n + \tfrac{b}{2}n^2 + \tfrac{m_\varphi^2}{2}\varphi^2 + \tfrac{\lambda_\varphi}{4}\varphi^4 + \tfrac{u}{2}n\varphi^2.$$

$a(T)$ tracks temperature T through the superconducting transition. $b > 0$ and $\lambda_\varphi > 0$ set the self-interactions. u ties the mode to the condensate. Its small-amplitude curvature becomes $m_\varphi^2 + un$. For a fixed dynamical normalization this predicts a shift of squared mode frequency with condensate density. The symbols φ and s describe different proposals. The model requires an independently measured mode and coupling. Magnetic levitation and position locking of a type-II superconductor have a conventional magnetic mechanism through screening currents and flux pinning. Neither a resonance nor this free energy alone predicts a change in gravitational acceleration.

14 How the wider framework connects

Dimensions constrain possible unifications. In four spacetime dimensions, with canonical Dirac and Maxwell fields and natural units, the powers of mass are $[\psi] = 3/2$, $[F] = 2$, $[S] = 3$ and $[X] = 6$. Every term in $\mathcal{L}$ must have mass dimension four. The direct action therefore requires $[\alpha_J] = -1$, $[\lambda_S] = -8$ and $[\Phi_0] = 3$. The direct manuscript calls α dimensionless. To give it that meaning one must supply an inverse mass scale, for example $\alpha_J = c_P/M_*$ with c_P dimensionless. This check affects any attempt to use one coefficient for a material, a compact star and a galaxy.

The recovered *v150* excerpt records an interaction

$$-\frac{\kappa_{150}}{M_*^4}(\bar{\psi}\sigma^{\mu\nu}\psi)(\bar{\psi}\sigma_{\mu\nu}\psi)F_{\rho\sigma}F^{\rho\sigma}.$$

It has the structure of the leading composite exponential correction. The displayed operator has dimension ten, so this notation requires $[\kappa_{150}] = -2$. An equivalent dimensionally explicit coefficient is C_{10}/M_*^6 for dimensionless C_{10}, with sign absorbed by convention. M_* denotes a reference mass scale. Matching the normalization and sign to the exponential needs the original conventions.

Torsion, collapse and daughter universes. An Einstein–Cartan version promotes the connection to an independent variable. With one common convention its torsion is $T^{\lambda}{}_{\mu\nu} = \Gamma^{\lambda}{}_{\mu\nu} - \Gamma^{\lambda}{}_{\nu\mu}$. Varying the connection couples geometry to matter's *three-index* spin current. The two-index electromagnetic polarization $S_{\mu\nu}$ cannot be substituted for that source without a map. The master's illustrative dust-plus-negative-spin cosmology has scale factor $a(t)/a_b = [1 + (t/t_b)^2]^{1/3}$, with a_b its minimum and t_b its time scale. It shows a possible bounce under those assumptions. An astrophysical black-to-white-hole transition or daughter universe needs an interior solution, a horizon and matching to the exterior. The bounce example supplies none of those by itself.

Microscopic matching. The direct term has the form of a Pauli magnetic dipole interaction. The electroweak, Higgs and E_8 proposals concern where such operators and constants could come from. A unification would have to specify the particle representations, symmetry breaking, interaction coefficients and their low-energy values. Quantum entanglement, clock comparisons and proposed observer effects have their own observables in the archive. The available actions give no unique new numerical coefficient for those extensions. Distinguishing a source's stated ambition from an observable equation preserves both.

The open link in one calculation. The revised galaxy curve fixes what radial quantity would be needed: $(\beta_s/c^2)sJ_s^r = v_\infty^2 r/[4\pi G(r^2 + r_c^2)]$. If microscopic spin alignment and entropy actually drive the curve, an independent material and galaxy calculation should deliver that product, with its units, size and radial dependence, after the coefficients have been fixed elsewhere. That calculation would connect the original spin question to the rotation data.

15 Sources and notation cross-reference

Original ESFD materials. Christopher Grenke, June 3, 2025, *Spin–Field Saturation Dynamics*, sections 3.1–3.4 for the explicit action and conventions. The companion is transcribed from the same day's 23:38 UTC consolidation. It contains the compact ESFD base, SEFD Lagrangian, spin pseudovector and halo prescription. The earlier June merged text gives the exponential variant. The standalone spin–entropy Word paper and complete historical v150 canvas were not among the readable originals. The recovered v150 passage supplies only the operator reproduced here. June Word manuscript · Source and equation reconciliation

The project PDFs. *Proof of the Electromagnetic Spin–Field Dynamics (ESFD) Theory*, sections 1–2 and its strong-field discussion, gives the κ_P interaction, emergent-gravity sketch and particle critical-field ratio. *Re-Evaluation of Electromagnetic Spin–Field Dynamics (ESFD) Theory*, opening framework, uses χ as a coupling. *Testing Electromagnetic Spin–Field Dynamics (ESFD) Theory with Astrophysical Data*, pages 1–2, gives the stellar energy estimate. Proof manuscript

Project calculations and data. The November master, equations 16, 17, 55 and 57, supplies the entropy equation, radial flux, superconducting extension and cored galaxy law. The NGC 2403 plot uses both saved SPARC prediction tables, fold 2. The 131-galaxy results use one RMS per held-out galaxy. The table values and plotted columns were checked for this guide. Master research paper · Data and code package · Standing-wave calculation · Pulsed-superconductor study

External reference material. Lelli, McGaugh and Schombert (2016) describe SPARC and its galaxy mass models. David Tong's quantum-field-theory notes describe Lagrangians, Dirac fields and ordinary electrodynamics. CERN's superconductivity lecture describes vortices and flux pinning. The ESFD-specific actions and result statistics above come from the project sources. SPARC paper · Field theory · Maxwell equations · Superconductivity

Symbols that are easy to mix up

- $\mathcal{A}$ is an action. A_μ is an electromagnetic potential. A_r in the spherical halo derivation denotes the weighted radial flux.
- $S_{\mu\nu}$ is spin polarization. Lowercase s is the proposed entropy density. Ψ is a superconducting order parameter. ψ is the Dirac matter field.
- Φ_0 sets the saturation scale. Φ_{EM} is defined differently in two manuscripts. Φ_f is a Newtonian potential for the revised galaxy law.
- R in the action is spacetime curvature. R_* is a star's radius. R_d is a galaxy's stellar disk scale length. r_s sets the decline of the earlier flux.
- Λ_{cc} is a cosmological constant. Λ_{tim} normalizes $B\Omega$. Λ_s normalizes $s\Omega$. Their units and roles differ.